

How Do Adults Solve Digital Tangram Problems? Analyzing Cognitive Strategies Through Eye Tracking Approach

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Abstract. Purpose of the study is to investigate how adults solve tangram based geometry problems on computer screen. Two problems with different difficulty levels were presented to 20 participants. The participants tried to solve problems by placing seven geometric objects into correct locations. In order to analyze the process, the participants and their eye movements were recorded by an Tobii Eye Tracking device while solving the problems. The results showed that the participants employed different strategies while solving problems with different difficulty levels

Keywords: Tangram, problem solving, eye tracking, spatial ability.

1 Introduction

Spatial ability is directly related to individuals' daily life experiences. Therefore, using visualization based activities in science and mathematics curriculum is supported by educators. Olkun (2003) [10] identified spatial visualization as the mental manipulation of objects and their parts in two-dimensional and three-dimensional spaces. Other researchers define it as the ability to mentally restructure of manipulate the components of the visual stimulus and involves recognizing, retaining and recalling configurations when the figure or parts of the figure are moved [4]. Linn and Petersen (1985) defined spatial ability as representing, transforming, generating and recalling symbolic, nonlinguistic information (pp.1482) [8]. They classified three types of spatial ability: mental rotation, spatial perception and spatial visualization. They defined mental rotation as ability to rotate a two or three-dimensional figure rapidly and accurately [3]. Spatial perception related to orientation of an individual's own body and high spatial perception requires overcoming distracting information [7]. Spatial visualization involves complicated, multistep manipulation of spatially presented information [7].

Tangram, a well-known Chinese puzzle, allows people form over 1000 patterns from seven geometric pieces. Indeed, in Mathematics, tangram is used to help learners to develop their spatial abilities [10], [11]. Kennedy and Tipps (1994) argued that

materials made even the most difficult mathematical concepts easier to understand. In addition, they are more understandable for the learners specially bridging between abstract concepts to real objects. Tooke, Hyatt, Leigh, Snyder and Borda (1992) [12] claimed that Mathematics is better learned when the learners have more experiences in manipulations. As stated by Ben-Chaim, Lappan and Houang (1989), [2] visualization provide the learners additional strategies potentially enriching their problem solving repertoire.

In our study, the term spatial ability is used to describe mental manipulation of 2D geometric objects including activities such as rotation of geometric objects, perceiving the geometric objects' space (big or small) and creating new geometric objects by combination of other figures. The assigned problem solving task is to create a tangram object with seven geometric objects. Problem solving is defined as any goal directed sequence of cognitive operations [1]. The main goal behind the idea of problem solving is to seek different solution angles within the problem solver's knowledge which is basically constituted in the memory [5]. This study aims to explore adults' 2D problem solving abilities with digital tangram. The main research question with the sub questions is,

How do participants solve digital tangram problems with different difficulty levels?

- ❖ How do eye fixation durations of the participants differ according to difficulty levels of the geometric figures?
- ❖ How do the participants' eye fixation counts change according to difficulty levels of the geometric figures?
- ❖ How do the participants' task completion durations change according to difficulty levels of the geometric figures?
- ❖ How do the participants' transition numbers between screens change according to difficulty levels of the geometric figures?
- ❖ What are the behaviors patterns of the participants while they are solving the digital problems?

2 Methodology

Twenty graduate students, between 20 to 30 years of age, participated to this study. They used digital tangram software to solve two different problems, with different complexity levels. Participants were first allowed to play with an easy figure to become familiar with the software. After they felt comfortable with the controls, they proceeded to the actual tasks.

The participants were asked to make patterns like a bird and crow. (Fig. 1. Level 1: First problem and geometric objects to solve puzzles. Level 2: Second problem and geometric objects to solve puzzles. level 1 and level 2). For the easy puzzle (level 1), the placement of the pieces is discernible and rotating some of the pieces is required. For the more difficult level (level 2), the places of the pieces are not so obvious when

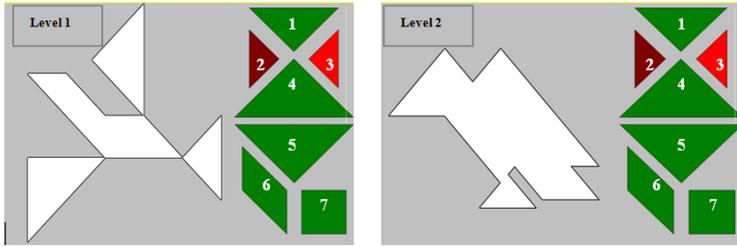


Fig. 1. Level 1: First problem and geometric objects to solve puzzles. Level 2: Second problem and geometric objects to solve puzzles.

compared to the easy one. We defined two sections on the screen under the Area of Interests (AOIs) category. The first part, left side of the screen which is named after ‘problem part’, consists of a specific figure that the participants have to create. The second part, right part of the screen, which is named after ‘geometric objects’, consists of different geometric objects (triangle, square, parallelogram) to create the certain figure on the pattern screen. Geometric objects part was numbered by the researchers to be used in the result section.

The participants and their eye movements were recorded by a Tobii Eye Tracking device while solving the problems. In this study, we had two independent variables ‘Figure complexity’ (level 1 and 2) and ‘Screen Differences’ (problem and geometric objects screens). In addition there were five dependent variables; ‘Time to First Fixation’, ‘Fixation count (number of fixations)’, ‘Average fixation duration (millisecond)’, ‘Transition (number of transitions)’ and ‘Task completion duration (seconds)’. Time to first fixation is defined as time from the beginning of the recording until the respective AOIs were first fixated upon. Fixation count is defined as number of fixations in the respective AOIs. Average fixation duration is defined as the average length of all fixations during all recordings on the respective AOIs. Transition number is the number of eye switching between the two sections of the screen (from problem screen to geometric objects screen and vice versa. Task completion duration is the time that the participants completed the task with success.

3 Results

3.1 Differences Between Level 1 and Level 2

It was investigated whether or not there is a significant difference between different difficulty levels of problems. The results showed that there is a significant difference between two levels, $t(38)=-2.794$, $p=.01$ for fixation count, $t(38)=-2.914$, $p=.008$ for task completion duration and for transition number, $t(38)=-2.037$, $p=.049$. However, there is no significant difference for average fixation duration and time to first fixation (Table 1).

Table 1. Mean score differences between level 1 and 2

		Minimum	Maximum	Mean	Std. Deviation
Level 1	Task completion	18.0	145.0	62.7	32.6
	Transfer number	5.0	115.0	36.9	29.9
	Time to 1 st fixation	478.0	5075.0	2021.7	1227.0
	Average fixation duration	166.0	437.0	297.2	75.4
	Fixation count	15.0	185.0	68.7	43.8
	Transitions	5.0	115.0	36.9	29.9
Level 2	Task completion	27.0	329.0	128.6	95.7
	Transfer number	10.0	228.0	67.3	59.6
	Time to 1 st fixation	383.0	8032.0	2117.4	1865.3
	Average fixation duration	152.0	420.0	286.1	62.5
	Fixation count	20.0	404.0	148.4	119.7
	Transitions	10.0	228.0	67.3	59.6

3.2 Differences Between Problem and Geometric Objects Screens

The results were investigated whether there is a significant difference between participants’ focus on the problem and geometric objects Area of Interest (AOI) screens according to their fixation count, average fixation duration and time to first fixation (Table 2).

For the level 1, there is a significant difference between problem and geometric objects screens based on participants’ fixation count, average fixation duration , $t(38) = 4.28, p = .000, t(38) = 3.06, p = .004$ respectively.

For the level 2, there is a significant difference between problem and geometric objects screens based on participants’ fixation count, average fixation duration $t(38) = 3.98, p = .001, t(38) = 3.57, p = .001$, respectively.

When we investigated the level differences for the problem screen, there is a significant difference between level 1 and 2 based on fixation count and gaze duration, $t(38) = -2.96, p = .007, t(38) = -3.11, p = .005$ respectively. However, there is no significant difference between level 1 and 2 based on participants’ average fixation duration for the problem screen. In detailed, when mean scores were investigated it was clear that mean of fixation count has increased from level 1 to level 2.

Additionally, when we examined the level difference on the geometric objects screen, there is no significant difference between level 1 and level 2 for participants’ fixation counts and average fixation durations. However, it can be seen that the mean scores of the participants’ fixation count has increased from level 1 to level 2 for the geometric objects screen.

Moreover, for verifying the data, the hotspot data were also investigated (Fig. 2). Analyzing the hot spots of the screen is a powerful technique to understand the gaze behavior and for better visualization of the eye movements of the participants. The most focusing area was colored as red while others green.

Table 2. Mean score differences between problem and geometric objects screens

		Screen	Mean	Std. Deviation
Level 1	Fixation count	problem	103.1	65.2
		geometric objects	35.1	27.9
	Average fixation duration	problem	342.1	118.1
		geometric objects	252.1	57.2
Level 2	Fixation count	problem	238.7	193.9
		geometric objects	58.6	56.9
	Average fixation duration	problem	331.4	99.8
		geometric objects	241.0	52.9

When we investigated the hotspots of the first problem (a bird), the participants focused on the head of the bird composing a small triangle and a parallelogram. Moreover, it was clear that participants tended to focus on the problem screen rather than the geometric objects screen for this level.

For the second problem (a crow), the hotspots of the participants showed that they focused on the discernible pieces than the not obvious ones. For this figure, the most fixated place was the leg of the crow and similarly for level 1, participants tended to focus on the problem part than the geometric objects. However, when the two levels were analyzed together in terms of the focusing areas of the participants, we see that when the complexity level increases the participants had a tendency to focus on the problem screen more than the geometric objects screen.

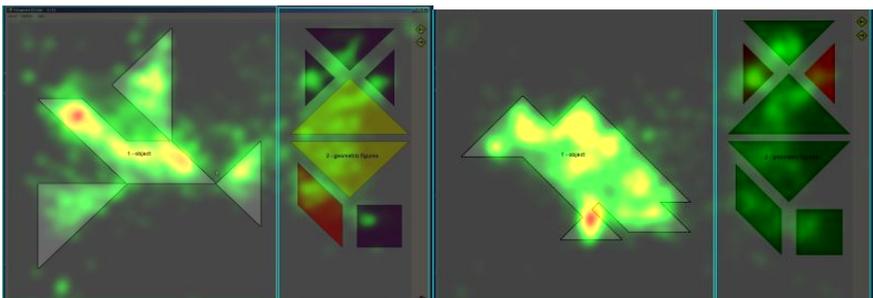


Fig. 2. The participants' focusing places on the level 1 and 2

3.3 Behavior Patterns for Level 1

Gaze replies of the participants were watched and investigated to understand how they had been solved the first problem. Figure 3 shows how the participants placed seven geometric objects in seven steps. Initially, the participants began to place from

big and discernible parts than the inconspicuous ones. That is most of the participants placed geometric objects 4 and 5 (Fig. 1) in the beginning. Another explicit result was that the parallelogram was placed in the fourth step. The participants preferred to place small parts after most of the figures were placed. For example, most of the participants placed the object 4 at the first step. This may show that participants generally focused on the particular objects rather than the overall picture. Parallel to this findings, most of the participants placed the object 3 (small object) at the seventh step. Also, the object 7 was never used in step1 and 2. This behavior pattern of the participants may be interpreted as an inductive approach because they solved the problem by mainly focusing on particular objects rather than general figure (Fig.3).

The maximum time required to solve the problem was 145 seconds (see Table 1), nine out of 20 participants made mistakes while solving the problem. A total of 11 mistakes were done. These participants generally put some figures to wrong places. The parallelogram had been the most difficult figure and most participants could not place this part in the first try.

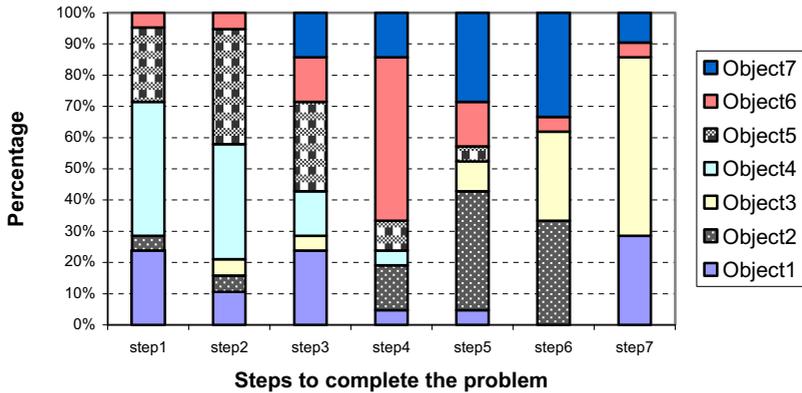


Fig. 3. Behavior patterns for the level 1

3.4 Behavior Patterns for the Level 2

The participants solved the problem 2 in three different ways (Fig. 4). In detail, 10 participants solved it by the strategy 1 and eight participants solved by the strategy 2. The remaining two participants solved the problem by the strategy 3.

For the second problem, a specific behavior pattern could not be observed. Its complex nature made more than one solution possible. However, the researchers observed the number of steps it took to solve the problem. According to the results, the participants solved the problem in 15 steps on the average. Maximum step count to solve problem was 34 and minimum was seven. Only three participants solved the problem in seven steps with no mistake. Furthermore, the most difficult objects were big triangles (geometric objects 4 and 5, see Fig. 1) and participants tried 51 times to find the correct place for these geometric objects. After these geometric objects the most difficult objects to place were geometric objects 1 and 7. Especially, the geometric object 7 (the square) was the hardest handled piece. It was observed that

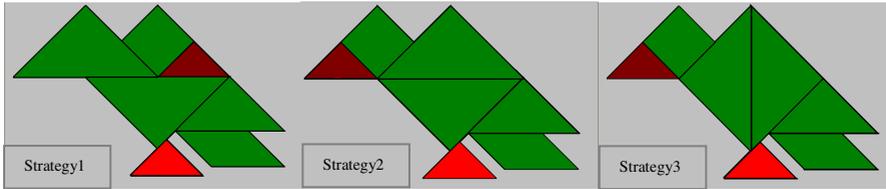


Fig. 4. Three solution strategy for the level 2

the participants had difficulties to rotate this geometric object. They needed to rotate the square 45° clockwise for placing this geometric object properly. It was observed that the participants realized it later than the placement of other geometric objects.

4 Conclusion and Discussion

Eye-tracking data showed that participants tended to choose different strategies while solving problems with different difficulty levels. The results showed that there are significant differences between level 1 and level 2 in accordance with the task completion duration and transition number. These results are interpreted as there might be a relationship among problem solving process, complexity level of the task and number of eye transitions between screens. As an expected result of this study, in case of the complexity level increase, the problem solving process is affected and fixation count, task completion duration and transition number are increased, too. Also, the increase in transition number increase may be interpreted as a mental process emerged and in the problem solving process the increase of complexity level is directly related with the task completion duration.

According to the AOIs results, the participants focused especially on the problem screen rather than on the geometric objects screen. In addition, when it was compared the level 1 to level 2 problem screens, the participants focused on the problem screen especially for the level 2'. When the two levels of the geometry problems were investigated, the reason for finding difference between the problem and geometric objects screen among participants may be related with the tendency to focus on the problem part for finding a solution. Additionally, as in the Table 2, when the complexity level increases, the number of focusing on the problem screen was increased, too. This is obvious that the participants inclined to focus on the problems screen as the complexity level increased. Although the results indicated that there was not a significant difference between level 1 and level 2 in terms of the focusing on geometric objects screen, hotspot data showed that participants focused more on level 2's geometric objects part than on the level 1's. Furthermore, there was a significant difference between participants in terms of focusing on problem screen between the first and second problems. It was clear that participants generally focused on the problem part comparing with the geometric objects when the complexity level is increased. Learners generated different strategies for different problems like inductive or deductive approaches. In problem 1, it was easy to recognize the objects rather than the overall picture because of the discernible parts in the problem. For this reason, they had a tendency to place the bigger and discernible objects before the small

objects. However, in the second problem the participants needed to think about the relationship between objects and their places for comprehending the whole picture and solve the problem. For this reason, they tried to see the overall picture rather than focusing on individual objects. This may be an evidence that the participants had chosen to follow a deductive approach.

Also, there might be a relationship between complexity level and the individuals' cognitive process like formation of new strategies. Participants may choose deductive strategies for the complex problem and they may choose strategies that are more inductive for the easier problems. This might be a clue to understand learners' problem solving strategies and whether there is a relationship between the task complexity and focus place of the participants on the screen.

As seen in the behavior pattern data and hotspot data, the participants tended to focus on the big and discernible objects than the vague ones. This might be an evidence for describing the problem solving process in terms of putting the objects in correct position since the participants placed the discernible objects firstly and inconspicuous ones later than the other objects. Also, this result is supporting the reason behind placing parallelogram lastly as its position was not clearly seen. Almost all participants had difficulty while placing the square and the reason that lay beneath might be related to the rotation problems of this object.

While solving the problem in the level 2, it was investigated that the participants followed three different strategies. In the first steps of the problem, the participants tended to place the discernible and big objects in the screen like the problem in level1. However, as it was seen in the findings, the participants had difficulties while placing these big objects that they had tried to place the objects 4 and 5 for 51 times. The participants might carry the first problem solving strategy that they used in level1 to the problem in level2. However, when they examined that the solution was not efficient, they changed their strategy and developed different strategies for solving the problem.

People have a tendency to use the familiar strategies that they used before, in the process of solving a problem but these previously known solving strategies may not be efficient in every condition. For this reason, it should be reminded by the educators to try different solution strategies. Also, in this process educators need to give some clues about developing strategies in regards of problem solving. Lastly, if digital tangrams are considered to be used in educational settings, as much as possible various solving strategies have to be presented to learners. The more diverse examples are given, the more successful applications are solved with tangrams in terms of accustoming to the problems.

5 Limitations and Future Studies

In this study, the sample size of the study is at the lowest limit of experimental studies. Therefore, this study may be repeated with a wider sampling especially with participants from diverse backgrounds. Furthermore, in another study, children needs to participate to the study for generalizing the results among different age groups and this may help to the educators in terms of using the digital tangrams in the educational

platforms effectively. Moreover, it can be investigated what the possible differences are between participants in regards to use digital tangrams and hand made tangrams.

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